Mark Scheme (Results)

Summer 2013

International GCSE Further Pure Mathematics Paper 1 (4PM0/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- **Types of mark**
    - M marks: method marks
    - A marks: accuracy marks. Can only be awarded if the relevant method mark(s) has (have) been gained.
    - B marks: unconditional accuracy marks (independent of M marks)
- **Abbreviations**
    - cao – correct answer only
    - ft – follow through
    - isw – ignore subsequent working
    - SC – special case
    - oe – or equivalent (and appropriate)
    - dep – dependent
    - indep – independent
    - eeoo – each error or omission
- **No working**

  If no working is shown then correct answers may score full marks.

  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**

  If there is a wrong answer indicated always check the working and award any marks appropriate from the mark scheme.

  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.

  Any case of suspected misread which does not significantly simplify the question loses two A (or B) marks on that question, but can still gain all the M marks. Mark all work on follow through but enter A0 (or B0) for the first two A or B marks gained.

  If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.

  If there are multiple attempts shown, then all attempts should be marked and the highest score on a single attempt should be awarded.

  In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme

- **Follow through marks**

  Follow through marks which involve a single stage of calculation can be awarded without working since you can check the answer yourself, but if ambiguous do not award.

  Follow through marks which involve more than one stage of calculation can only be awarded on sight of the relevant working, even if it appears obvious that there is only one way you could get the answer given.

- **Ignore subsequent working**

  It is appropriate to ignore subsequent working when the additional work does not change the answer in a way that is inappropriate for the question: e.g. incorrect cancelling of a fraction that would otherwise be correct.

  It is not appropriate to ignore subsequent working when the additional work essentially shows that the candidate did not understand the demand of the question.

- **Linear equations**

  Full marks can be gained if the solution alone is given, or otherwise unambiguously indicated in working (without contradiction elsewhere). Where the correct solution only is shown substituted, but not identified as the solution, the accuracy mark is lost but any method marks can be awarded.

- **Parts of questions**

  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded in another.

General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x =

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x =

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c, \quad q \neq 0,$ leading to x =...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1.

2. Integration:

Power of at least one term increased by 1.

Use of a formula:

Generally, the method mark is gained by **either** quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values **or**, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks". General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show....")

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Question Number	Scheme			Marks
	(a)	(b)	(c)	
1.	$126 = \frac{1}{2}12^2\theta$	*or*	$\frac{\theta}{360}\times\pi\times12^2 = 126$	M1
	$\theta = \frac{126}{72} = 1\frac{3}{4}$	$126 = \frac{1}{2}\times12\times l$	$\theta = \frac{126\times360}{144\pi} = 100.27^\circ$	A1
	$l = 12\times\frac{7}{4}$	$l = \frac{126}{6}$	$l = \frac{100.27}{360}\times2\pi\times12 = \frac{126\times24}{144}$	M1
	= 21 (cm) Method (d) in Notes			A1 (**4**)

Notes

Question 1

Method (a) and (c)

M1 for an expression in either degrees or radians using A=126 to find angle θ

A1 for a fully correct expression with correct numerical values

M1 for an expression in either degrees or radians with their θ to find arc length AB

A1 AB = 21(cm) cso

Method (b)

M1 for a correct formula $\frac{1}{2}rl$

A1 for correct substitution of the value of r, (=12)

M1 for equating their formula to 126 cm^2

A1 = 21 (cm) cso

Method (d)

M1 for an area of a circle divided by 126

A1 for using $r = 12$

M1 for the length of the circumference of the circle divided by their value of the scale factor using a value for r of 12 only.

A1 for 21 (cm) cso

Note: Correct solution only seen – award full marks Allow 21.0 (cm)

Question Number	Scheme	Marks
2.	$3(x^2+2x+1)<9-x$ $3x^2+7x-6<0$ $(3x-2)(x+3)<0$ $-3<x<\frac{2}{3}$	 M1 A1 M1 A1 (4)

Notes

Question 2

M1 for obtaining a 3TQ equation or expression (=0 not required for this mark)

A1 for attempting to find their critical values as far as $x = \ldots$ (We are treating this as an M mark)

M1 for choosing the **inside** region for their critical values.

A1 cao for $-3<x<\dfrac{2}{3}$. Accept $-3<x$ **and** $x<\dfrac{2}{3}$ and $-3<x \cap x<\dfrac{2}{3}$.

Do not accept $-3<x$ **or** $x<\dfrac{2}{3}$, or $-3<x$ **,** $x<\dfrac{2}{3}$ .These are all A0

Use of $\leq$ loses the final A mark

Question Number	Scheme	Marks
3.	(a) $a=-3$ $\quad b=1$	B1 B1
	(b) at $(1,0)$ $\quad 0=1+\frac{c}{1-3}$	M1
	$-1=\frac{c}{-2}$ $\quad c=2$	A1
	at $(0,d)$ $\quad d=1+\frac{2}{-3}$	M1
	$d=\frac{1}{3}$	A1 (6)

Notes

Question 3

(a)

B1 for either *a* or *b*

B1 for both *a* and *b*

M1 for substituting in $y = 0$ and $x = 1$ into the equation of the curve. a need not be substituted for this mark

(b)

A1 for $c = 2$ cso

M1 for substituting $x = 0$ and $y =$ d into the equation of the curve to find *d*. Neither *c* nor *a* need to be substituted for this mark.

A1 $d = \frac{1}{3}$ cso.

Question Number	Scheme	Marks
4.	$6(1-\cos^2 x)-\cos x-4=0$	M1
	$6\cos^2 x+\cos x-2=0$	A1
	$(3\cos x+2)(2\cos x-1)=0$	M1
	$(\cos x=-\frac{2}{3})$ or $\cos x=\frac{1}{2}$	A1
	$x=-60$ or $x=60$	A1 A1 (6)

Notes

Question 4

M1 for using $\cos^2 x+\sin^2 x=1$ to achieve an equation in terms of cos x only. (=0 not required for this mark)

A1 for forming the correct 3TQ

M1 for solving their 3TQ as far as cos x = ……….(usual rules for an attempt) Their quadratic need not = 0 at this stage

A1 $\cos x=\frac{1}{2}$, ($\cos x=-\frac{2}{3}$ – this need not be seen)

A1 for either value of $x=$ 60 , $x=$ 60

A1 for both values $x=$ 60 $x=$ 60

If other values are given, ignore if not in range. Deduct one A mark for each extra value that is in range, up to a maximum of the last two A marks.

Question Number	Scheme	Marks
5.	$V=500 \Rightarrow 4h^3=500$	M1
	$\Rightarrow h=5$	A1
	$\dfrac{\mathrm{d}V}{\mathrm{d}h}=12h^2$	M1 A1
	$\dfrac{\mathrm{d}h}{\mathrm{d}t}=\dfrac{\mathrm{d}h}{\mathrm{d}V}\times\dfrac{\mathrm{d}V}{\mathrm{d}t}=\dfrac{1}{12h^2}\times 36$	M1
	$=\dfrac{36}{12\times 5^2}=\dfrac{3}{25}=0.12$ cm/s	M1 A1 (7)

<u>Notes</u>

Question 5

Note: Parts of the question can be found anywhere in their working on the page

M1 for $V=500 \Rightarrow 4h^3=500$

A1 $h=5$ cso

M1 for differentiating $V=4h^3$ (usual rules apply)

A1 for $\dfrac{dV}{dh}=12h^2$ cso

M1 for applying chain rule to find an expression for $\dfrac{dh}{dt}=\dfrac{dh}{dV}\times\dfrac{dV}{dt}$ or any correct arrangement (expression is sufficient – substitution of values is not required for this mark)

M1 for substituting values into their $\dfrac{dh}{dt}$

A1 for $\dfrac{dh}{dt}=\dfrac{3}{25}=0.12$ (cm s^{-1}) oe - exact answer only.

Question Number	Scheme	Marks
6.	(a) (i) $\alpha+\beta=-p$	B1
	(ii) *or* $\begin{cases}\alpha^2+p\alpha+1=0\\ \beta^2+p\beta+1=0\end{cases}$	
	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $\alpha^2+\beta^2+p(\alpha+\beta)+2=0$	M1
	$=p^2-2$ $\alpha^2+\beta^2=p^2-2$	A1
	(iii) $(\alpha+\beta)^3=\alpha^3+3\alpha^2\beta+3\alpha\beta^2+\beta^3$	M1
	$\Rightarrow\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=(-p)^3-3(-p)$	M1 A1
	$=3p-p^3$	
	alternatives $\begin{cases}\alpha^3+p\alpha^2+\alpha=0\\ \beta^3+p\beta^2+\beta=0\end{cases}$	
	$\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ $\alpha^3+\beta^3+p(\alpha^2+\beta^2)+(\alpha+\beta)=0$ M1	
	$=-p(p^2-2-1)$ $\alpha^3+\beta^3+p(p^2-2)-p=0$ M1	
	$=3p-p^3$ $\alpha^3+\beta^3=3p-p^3$ A1	
	(b) $x^2-(3p-p^3)x+1=0$	M1ft A1ft
		(8)

Notes

Question 6

(a) (i) B1 for $\alpha+\beta=-p$ or $\left(-\frac{p}{1}\right)$

(Note $\alpha\beta=1$)

(ii) M1 for $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ and substituting in values for $\alpha+\beta$, and $\alpha\beta$

Or for $\begin{cases}\alpha^2+p\alpha+1=0\\ \beta^2+p\beta+1=0\end{cases}$

$\Rightarrow\alpha^2+\beta^2+p(\alpha+\beta)+2=0$

A1 for $\alpha^2+\beta^2=p^2-2$ oe (Simplification is not required for this mark)

(iii) M1 for expanding $(\alpha+\beta)^3=\alpha^3+3\alpha^2\beta+3\alpha\beta^2+\beta^3$ (allow some slips in algebra for this mark). Do **NOT** accept $(\alpha+\beta)^3=\alpha^3+\beta^3$ for this mark

M1 leading to $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ fully correct

A1 for $\alpha^3+\beta^3=3p-p^3$ oe (Simplification is not required for this mark)

Please refer to ms for alternative methods

(b)

M1 for using x^2 – their sum × x + product (= 0 not needed for this mark)

`A1ft for $x^2-(3p-p^3)x+1=0$ (follow through their values for this mark)

Note: = 0 must be seen with a correct equation for this mark

Simplification is not required for this mark

Question Number	Scheme	Marks
7.	(a) (i) $t_{58} = a + 57d$	B1
	(ii) $S_{13} = \frac{13}{2}(2a + 12d)$	B1
	(b) $a + 57d = \frac{13}{2}(2a + 12d)$	
	$-12a = 21d$	M1
	$d = -\frac{4}{7}a$	A1
	(c) $t_{176} = a + 175d = a + 175(-\frac{4}{7}a)$ OR $S_{21} = \frac{21}{2}(2a + 20d) = 21a + 210(-\frac{4}{7}a)$	M1
	$= a - 100a = -99a$	A1
	$S_{21} = \frac{21}{2}(2a + 20d) = 21a + 210(-\frac{4}{7}a)$ OR $t_{176} = a + 175d = a + 175(-\frac{4}{7}a)$	M1
	$= 21a - 120a = -99a = t_{176}$	A1
	(d) $a + (r-1)d = 5(a + 8d)$	M1
	$(r-1)d = 4(-\frac{7}{4}d) + 40d$ or $(r-1)(-\frac{4}{7}a) = 4a + 40(-\frac{4}{7}a)$	
	$r - 1 = 33$ or $-4(r-1) = -132$	M1
	$r = 34$	A1 (11)

<u>Notes</u>

Question 7(a)

(i) B1 for any correct expression for t_{58} (simplification not required for this mark)

(ii) B1 for any correct expression for S_{13} (simplification not required for this mark)

(b)

M1 for their t_{58} = their S_{13}

A1 for collecting like terms on either side leading to $d = -\frac{4}{7}a$ cso *

This is a 'show' question so all working must be seen clearly.

(c)

M1 for an expression for t_{176} or S_{21} in either a or d

Substitution must be for the **given** value of d

A1 for $t_{176} = a - 100a = -99a$ or $t_{176} = \frac{693}{4}d$ cso OR $S_{21} = 21a - 120a = -99a$

M1 for an expression for t_{176} or S_{21} in either a or d

Substitution must be for the **given** value of d

A1 for $t_{176} = a - 100a = -99a$ OR $S_{21} = 21a - 120a = -99a = t_{176}$ or $t_{176} = S_{21} = \frac{693}{4}d$ cso **with a conclusion**

Alternative

M1 for $t_{176} = S_{21}$ using 'their' expressions

A1 for correct unsimplified $t_{176} = S_{21}$

M1 for $-35d = 20a$ oe

A1 for $d = -\frac{4}{7}a$ with a conclusion that must refer to part (b)

(d)

M1 for equating expressions for t_r and $5t_9$ in r, a and d

M1 for an equation in r only (allow for slip ups in algebra for this mark)

A1 $r = 34$ cso

Question Number	Scheme	Marks
8.	(a) $15+2x-x^2=0$	M1
	$(5-x)(3+x)=0 \Rightarrow x=5, x=-3$	M1 A1
	(b) $\int_{-3}^{5}(15+2x-x^2)\mathrm{d}x$	M1
	$=\left[15x+x^2-\frac{1}{3}x^3\right]_{-3}^{5}$	A1
	$=(75+25-\frac{125}{3})-(-45+9+9)$	M1
	$=85\frac{1}{3}$	A1
	(c) $x+9=15+2x-x^2$	M1
	$x^2-x-6=0 \Rightarrow (x-3)(x+2)=0 \Rightarrow x=3,\ x=-2$	M1 A1
	(d) $M=85\frac{1}{3}-\int_{-2}^{3}\left\{(15+2x-x^2)-(x+9)\right\}\mathrm{d}x$	M1
	$=85\frac{1}{3}-\int_{-2}^{3}\left\{6+x-x^2\right\}\mathrm{d}x$	
	$=85\frac{1}{3}-[6x+\frac{1}{2}x^2-\frac{1}{3}x^3]_{-2}^{3}$	A1
	$=85\frac{1}{3}-\left\{(18+4\frac{1}{2}-9)-(-12+2+\frac{8}{3})\right\}$	M1
	$=85\frac{1}{3}-20\frac{5}{6}=64\frac{1}{2}$	A1 (14)
	Alternative	
	(d) $M=\int_{-3}^{-2}(15+2x-x^2)\mathrm{d}x+\frac{1}{2}(7+12)5+\int_{3}^{5}(15+2x-x^2)\mathrm{d}x$ M1	
	$=[15x+x^2-\frac{1}{3}x^3]_{-3}^{-2}+\frac{95}{2}+[15x+x^2-\frac{1}{3}x^3]_{3}^{5}$ A1	
	$=(-30+4+\frac{8}{3})-(-45+9+9)+47\frac{1}{2}+(75+25-\frac{125}{3})-(45+9-9)$ M1	
	$=3\frac{2}{3}+47\frac{1}{2}+13\frac{1}{3}=64\frac{1}{2}$ A1	

Notes

Question 8

(a)

M1 for setting $15+2x-x^2=0$

M1 for solving the quadratic as far as $x=$ ……

A1 for $x=5,\ x=-3$

(b)

Ignore limits for **first** M1 and A1

M1 for an attempt at $\int_{-3}^{5} 15x+2x-x^2 dx$ (Usual rules) ft their values of x in (a)

A1 for a fully correct integrated expression

M1 for an evaluation of their integrated expression with their limits

A1 for an area = $85\frac{1}{3}$ or $\frac{256}{3}$ or awrt 85.33 (with a **minimum** of 2dp) cso.

(c)

M1 for equating line l with curve C ($x+9=15+2x-x^2$)

M1 for forming a 3TQ and attempting to solve as far as $x=$

A1 for $x=3,\ \ x=-2$

(d)

M1 for forming a COMPLETE expression of the area, either from,

$$\text{M} = 85\frac{1}{3} \text{ (or their area in part (b))} - \int_{-2}^{3}\left\{(15+2x-x^2)-(x+9)\right\}\mathrm{d}x$$

or, $M=\int_{-3}^{-2}(15+2x-x^2)\mathrm{d}x+\frac{1}{2}(7+12)5+\int_{3}^{5}(15+2x-x^2)\mathrm{d}x$

using their limits found in (c)

A1 for correct integration of their expression for the area

dM1 for evaluating their integrated expression for the area

A1 either, $=85\frac{1}{3}-20\frac{5}{6}=64\frac{1}{2}$, or $=3\frac{2}{3}+47\frac{1}{2}+13\frac{1}{3}=64\frac{1}{2}$ oe – **exact answer only**

NOTE: If they do not form a **complete** expression for the area, then M0 A0 dM0 A0

Question Number	Scheme	Marks
9.	(a) $\angle ABC = 90^\circ$	B1
	$\cos 30 = \frac{BC}{12}$ or $\sin 60 = \frac{BC}{12}$	M1
	$BC = 12\cos 30 = 6\sqrt{3}$ cm or $BC = 12\sin 60 = 6\sqrt{3}$ cm	A1
	(b) $\sin 30 = \frac{BP}{6\sqrt{3}}$	M1
	$\Rightarrow BP = 6\sqrt{3}\sin 30 = 6\sqrt{3}\times\frac{1}{2} = 3\sqrt{3}$ cm	A1
	(c) $\tan 25 = \frac{3\sqrt{3}}{BF}$ or $\tan 65 = \frac{BF}{3\sqrt{3}}$	M1
	$\Rightarrow BF = \frac{3\sqrt{3}}{\tan 25}$ or $BF = 3\sqrt{3}\tan 65$	A1
	$\Rightarrow BF = 11.1$ cm (3SF)	A1
	(d) $BD^2 = (3\sqrt{3}\tan 65)^2 + (6\sqrt{3})^2$ *or* $DP^2 = (3\sqrt{3}\tan 65)^2 + (3\sqrt{3}\tan 60)^2$	M1
	$BD = \sqrt{232.17} = 15.24$ *or* $DP = \sqrt{205.2} = 14.32$	A1
	$\sin BDP = \frac{3\sqrt{3}}{15.24}$ *or* $\tan BDP = \frac{3\sqrt{3}}{14.32}$	M1
	$\angle BDP = 19.9^\circ$	A1
	(e) Volume $= \frac{1}{2}\times 12\times 3\sqrt{3}\times(3\sqrt{3}\tan 65)$	M1
	$= 162\tan 65^\circ = 347\ \text{cm}^3$ (3SF)	A1 (14)

<u>Notes</u>

Please note the stipulations on exact answers and the rounding required. Please refer to General Principles.

Question 9

(a)

B1 for $\angle ABC = 90^\circ$, can be implied from working

M1 for any acceptable trigonometry using a complete method to find *BC*

A1 for the value $6\sqrt{3}$ only. Do not accept any decimal value for this mark

(b)

M1 for using any acceptable trigonometry using a complete method to find *BP*

A1 for the value of $3\sqrt{3}$ only * (this is a 'show' question, all working must be correct)

(c)

M1 for using any acceptable trigonometry using a complete method involving angles 25º or 65º

A1 for a correct expression for *BF*

A1 for *BF* = 11.1 (cm) – correct to 3sf for this mark

(d)

M1 for an attempt at an expression for *BD* or *DP*, please refer to the ms for examples - ft their values for *BC* and *BF*, but must use $3\sqrt{3}$ for *BP*

A1 for $BD = \sqrt{232.17} = 15.24$ or $DP = \sqrt{205.2} = 14.32$

M1 for using an expression of any acceptable trigonometry to find *BDP*

A1 for $\angle BDP = 19.9^\circ$ - correct to 1dp

(e)

M1 for an expression of the volume using the given *AC* (=12), $BP = 3\sqrt{3}$ only, and their *BF*

A1 for 347 cm^3 (correct to 3sf)

Lengths of line in the prism for examiners

AC = DE = 12

AB = EF = 6

BP = $3\sqrt{3}$

BF = CD = AE = 11.14……….

AD = CE = 16.37…………

CP = 9

AP = 3

BC = DF = $6\sqrt{3}$

DP = 14.32…….

BD = 15.24…………

Question Number	Scheme	Marks
10.	(a) $\frac{\mathrm{d}y}{\mathrm{d}x} = 4x^3 - 12x^2 - 4x + 13$	M1 A1
	at R, $\frac{\mathrm{d}y}{\mathrm{d}x} = 4 - 12 - 4 + 13 = 1$	
	l_1 has equation $y - 13 = 1(x - 1)$ $[y = x + 12]$	M1 A1
	(b) $4x^3 - 12x^2 - 4x + 13 = 1$	
	$4x^3 - 12x^2 - 4x + 12 = 0$	M1
	$4(x-1)(x^2 - 2x - 3) = 0$	
	$4(x-1)(x+1)(x-3) = 0$	M1
	$x = -1,\ x = 1,\ x = 3$	
	At P, $x = -1$, $y = 1 + 4 - 2 - 13 + 5 = -5$ so $P(-1, -5)$	A1
	At Q, $x = 3$, $y = 81 - 108 - 18 + 39 + 5 = -1$ so $Q(3, -1)$	A1
	(c) Gradient of $PQ = \frac{-5+1}{-1-3} = 1$	
	Equation of l_2 is $y + 1 = 1(x - 3)$ $[y = x - 4]$	M1 A1
	or $y + 5 = 1(x + 1)$	
	(d) Gradient of l_2 = gradient of C at P = gradient of C at Q [= 1]	B1
	[Since l_2 passes through P and Q with the same gradient as the curve at these points, it must be a tangent to C at P and at Q.]	
	(e) Normal at R has equation $y - 13 = -1(x - 1)$	
	At intersection with l_2, $(x-4) - 13 = -1(x - 1)$ or $y - 13 = -1(y + 4 - 1)$	M1
	$\Rightarrow 2x = 18$ or $2y = 10$	M1
	$\Rightarrow x = 9$ and $y = 5$	A1
	$RS^2 = (13-5)^2 + (1-9)^2$	M1
	$RS = \sqrt{64 + 64} = 8\sqrt{2}$	A1
	(f) $PQ = \sqrt{(-1-3)^2 + (-5+1)^2} = \sqrt{16+16} = 4\sqrt{2}$	
	Area $PQR = \frac{1}{2} \times 8\sqrt{2} \times 4\sqrt{2} = 32$	M1 A1 (18)
	alternative	
	(f) Area $PQR = \frac{1}{2}\begin{vmatrix} -1 & 3 & 1 & -1 \\ -5 & -1 & 13 & -5 \end{vmatrix} = \frac{1}{2}\left[(1 + 39 - 5) - (-15 - 1 - 13)\right]$ M1	
	$= \frac{1}{2}(35 + 29) = 32$ A1	

Notes
(a) M1 for an attempt at differentiation (usual rules – reducing the power of at least one term, the disappearance the constant is insufficient for this mark) A1 for a complete correct differentiated expression M1 for finding and using a numerical value of the gradient, derived only from using $\frac{dy}{dx}$ into either $y - 13 =$ (their m) $(x - 1)$, or by applying $y = mx + c$ including finding a value for c A1 for any correct equation $y - 13 = 1(x - 1)$ $\left[y = x + 12,\ \ y - x - 12 = 0\right]$ etc **(b)** M1 for setting their $\frac{dy}{dx} = 1$ and re-arranging to give a cubic equation (=0) M1 for factorising their equation leading to three values of x A1 for either of the correct coordinates $(-1, -5)$ **or** $(3, -1)$ ($x = -1$, $y = -5$ **or** $x = 3, y = -1$) A1 for both $(-1, -5)$ **and** $(3, -1)$ correct , ($x = -1$, $y = -5$ **and** $x = 3, y = -1$) **(c)** M1 for finding the numerical gradient of l_2 using their coordinates of P and Q, and attempting to form an equation using their gradient and the points P or Q A1 for a correct equation eg $y + 5 = 1(x + 1)$ or $y + 1 = 1(x - 3)$ $[y = x - 4]$ **(d)** B1 please refer to ms **(e)** M1 for forming the equation of the normal at R. They must use a numerical gradient derived from their gradient of the tangent in part (a) using the rule $m_t \times m_n = -1$, and use the given coordinate of R. $y - 13 = -1(x - 1)$ oe ($y = -x + 14$) M1 for finding the point of intersection of the Normal at R and l_2, by any acceptable method eg., simultaneous equations A1 for the point of intersection of S, either $x = 9$ and $y = 5$, or gives coords (9, 5) M1 for using Pythagoras with point R and their S A1 for $8\sqrt{2}$, $\sqrt{128}$ oe exact answer only **(f)** M1 for any method to find the area of triangle PQR ft their P and Q A1 for area $PQR = 32$ (units^2)

Question Number	Scheme	Marks
11.	(a) $\overrightarrow{AB}=2\mathbf{p}-2\mathbf{q}$ oe	B1
	(b) $\overrightarrow{BC}=6\mathbf{p}-4\mathbf{q}-(3\mathbf{p}-\mathbf{q})$ *or* $\overrightarrow{AC}=6\mathbf{p}-4\mathbf{q}-(\mathbf{p}+\mathbf{q})$	M1
	$=3\mathbf{p}-3\mathbf{q}$ $\quad$ $=5\mathbf{p}-5\mathbf{q}$	
	$\Rightarrow \overrightarrow{AB}=\frac{2}{3}\overrightarrow{BC}$ or $\overrightarrow{AB}\parallel\overrightarrow{BC}$ *or* $\Rightarrow \overrightarrow{AB}=\frac{2}{5}\overrightarrow{AC}$ or $\overrightarrow{AB}\parallel\overrightarrow{AC}$	A1
	$\Rightarrow A, B, C$ are collinear	
	(c) $AB : BC = 2 : 3$ oe	B1
	(d) $\overrightarrow{CD}=\frac{1}{2}\overrightarrow{AC}$ *or* $\overrightarrow{AD}=\frac{3}{2}\overrightarrow{AC}$ *or* $\overrightarrow{BD}=\frac{11}{10}\overrightarrow{AC}$	
	$=\frac{1}{2}((2\mathbf{p}-2\mathbf{q})+(3\mathbf{p}-3\mathbf{q}))$	M1
	$=\frac{1}{2}(5\mathbf{p}-5\mathbf{q})$ $\quad$ $=\frac{3}{2}(5\mathbf{p}-5\mathbf{q})$ $\quad$ $=\frac{11}{10}(5\mathbf{p}-5\mathbf{q})$	A1
	$\overrightarrow{OD}=(6\mathbf{p}-4\mathbf{q})+\frac{1}{2}(5\mathbf{p}-5\mathbf{q})$ *or* $(\mathbf{p}+\mathbf{q})+\frac{3}{2}(5\mathbf{p}-5\mathbf{q})$ *or* $(3\mathbf{p}-\mathbf{q})+\frac{11}{10}(5\mathbf{p}-5\mathbf{q})$	M1
	$=8\frac{1}{2}\mathbf{p}-6\frac{1}{2}\mathbf{q}$	A1 (8)
	alternative	
	$\dfrac{\overrightarrow{OA}+2\overrightarrow{OD}}{1+2}=\overrightarrow{OC}$ *or* $\overrightarrow{OD}=\dfrac{-\overrightarrow{OA}+3\overrightarrow{OC}}{-1+3}$ M1	
	$\dfrac{\mathbf{p}+\mathbf{q}+2\overrightarrow{OD}}{3}=6\mathbf{p}-4\mathbf{q}$ *or* $\overrightarrow{OD}=\dfrac{-(\mathbf{p}+\mathbf{q})+3(6\mathbf{p}-4\mathbf{q})}{2}$ A1	
	$2\overrightarrow{OD}=17\mathbf{p}-13\mathbf{q}$ *or* $\overrightarrow{OD}=\dfrac{17\mathbf{p}-13\mathbf{q}}{2}$ M1	
	$\overrightarrow{OD}=8\frac{1}{2}\mathbf{p}-6\frac{1}{2}\mathbf{q}$ A1	

Notes

Question 11

(a)

B1 for $\overrightarrow{AB} = 2\mathbf{p} - 2\mathbf{q}$ or any equivalent expression

(b)

M1 for finding a vector for $BC\ (= 3\mathbf{p} - 3\mathbf{q})$ **or** $AC\ (= 5\mathbf{p} - 5\mathbf{q})$

A1 for $\overrightarrow{AB} = \tfrac{2}{3}\overrightarrow{BC}$ or $\overrightarrow{AB} \parallel \overrightarrow{BC}$ **or** $\overrightarrow{AB} = \tfrac{2}{5}\overrightarrow{AC}$ or $\overrightarrow{AB} \parallel \overrightarrow{AC}$

So *A, B,C* are collinear cso – there must be two correct vectors

(c)

B1 for $AB : BC = 2 : 3$ (oe)

(d)

First Method

M1 for forming a vector equation for either *CD, AD,* or *BD*

A1 for $k\ (5\mathbf{p} - 5\mathbf{q})$ where k is either $\frac{1}{2}$, $\frac{3}{2}$ or $\frac{11}{10}$ for *CD, AD,* or *BD* respectively

M1 for finding an expression for *OD* (alternatives in ms)

A1 for $\overrightarrow{OD} = 8\tfrac{1}{2}\mathbf{p} - 6\tfrac{1}{2}\mathbf{q}$ oe

Second method

M1 for the ratio of AC: CD

A1 for AC:CD = 2 : 1

M1 for either component of **p** or **q** correct, ie., $8\frac{1}{2}\mathbf{p}$ OR $-6\frac{1}{2}\mathbf{q}$

A1 for a complete correct expression for , $\overrightarrow{OD} = 8\tfrac{1}{2}\mathbf{p} - 6\tfrac{1}{2}\mathbf{q}$, $\overrightarrow{OD} = \dfrac{17\mathbf{p} - 13\mathbf{q}}{2}$, oe

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