



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS

4037/01

Paper 1

For Examination from 2013

SPECIMEN MARK SCHEME

2 hours

MAXIMUM MARK: 80

This document consists of **6** printed pages.



Mark Scheme Notes

Marks are of the following three types:

- M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B Accuracy mark for a correct result or statement independent of method marks.
- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
 - The symbol \surd implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
 - Note: B2 or A2 means that the candidate can earn 2 or 0.
B2, 1, 0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts:

- AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
- BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
- CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
- ISW Ignore Subsequent Working
- MR Misread
- PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)
- SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

- MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{\quad}$ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.
- OW -1,2 This is deducted from A or B marks when essential working is omitted.
- PA -1 This is deducted from A or B marks in the case of premature approximation.
- S -1 Occasionally used for persistent slackness – usually discussed at a meeting.
- EX -1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

<p>1 (i) correct diagram</p> <p>(ii) correct diagram</p> <p>(iii) correct diagram</p>	<p>B1</p> <p>B1</p> <p>B1 [3]</p>	
<p>2 $(2x + 1)^2 > 8x + 9$ $4x^2 - 4x - 8 > 0$ $x^2 - x - 2 > 0$ $(x + 1)(x - 2) > 0$ Leads to critical values $x = -1, 2$ $x < -1$ and $x > 2$</p>	<p>M1</p> <p>DM1</p> <p>A1</p> <p>√A1 [4]</p>	<p>M1 for simplification to 3 term quadratic</p> <p>DM1 for factorisation</p> <p>A1 for critical values</p> <p>Follow through on their critical values.</p>
<p>3</p> $\text{LHS} = \frac{\sin^2 A + 1 + \cos^2 A + 2 \cos A}{(1 + \cos A) \sin A}$ $= \frac{2 + 2 \cos A}{(1 + \cos A) \sin A}$ $= \frac{2}{\sin A} \text{ leading to } 2 \cos e c A$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1 [4]</p>	<p>M1 for attempt to deal with fractions and attempt to obtain numerator</p> <p>A1 correct</p> <p>M1 for use of $\sin^2 A + \cos^2 A = 1$</p>
<p>4 Substitution of $x = 1$ leading to $a + b + 4 = 0$</p> <p>Substitution of $x = -\frac{1}{2}$ leading to $-a + 2b - 28 = 0$</p> <p>Leading to $a = -12, b = 8$</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 [5]</p>	<p>M1 for substitution of $x = 1$ and equated to 3</p> <p>M1 for substitution of $x = -\frac{1}{2}$ and equated to 6</p> <p>A1 for both correct</p> <p>M1 for solution</p> <p>A1 for both</p>
<p>5 (i) $2t^2 - 9t - 5 = 0$ $(2t + 1)(t - 5) = 0$ $t = \frac{1}{2}, t = 5$</p> <p>(ii) $x^{\frac{1}{2}} = -0.5, 5$ $x = 0.25, 25$</p>	<p>M1</p> <p>DM1</p> <p>A1 [3]</p> <p>M1</p> <p>A1, A1 [3]</p>	<p>M1 for attempting to form a quadratic in t</p> <p>DM1 for attempt to solve a 3 term quadratic</p> <p>A1 for both</p> <p>M1 for realising that $x^{0.5}$ is equivalent to t (or valid attempt at solution)</p>
<p>6 (i) $\mathbf{a} = \frac{1}{13} (5\mathbf{i} - 12\mathbf{j})$</p> <p>(ii) $q(5\mathbf{i} - 12\mathbf{j}) + p\mathbf{i} + \mathbf{j} = 19\mathbf{i} - 23\mathbf{j}$ $5q + p = 19$ $-12q + 1 = -23$ Leading to $q = 2, p = 9$</p>	<p>M1, A1 [2]</p> <p>M1</p> <p>M1</p> <p>A1 [3]</p>	<p>M1 for a valid attempt to obtain magnitude.</p> <p>M1 for equating like vectors</p> <p>M1 for solution of (simultaneous) equations</p> <p>A1 for both</p>

<p>7 (i) $y = 4x^2 - 12x + 3$ $y = (2x - 3)^2 - 6$</p> <p>(ii) $\left(\frac{3}{2}, -6\right)$</p> <p>(iii) $f \geq -6$</p>	<p>B1 B1 B1 [3]</p> <p>$\sqrt{B1}$, $\sqrt{B1}$ [2]</p> <p>$\sqrt{B1}$ [1]</p>	<p>B1 for 2 (part of linear factor) B1 for -3 (part of linear factor) B1 for -6</p> <p>Follow through on their a, b and c Allow calculus method.</p> <p>Follow through on their c</p>
<p>8 $\frac{dy}{dx} = -2e^{-2x}(+c)$</p> <p>When $\frac{dy}{dx} = 3, x = 0, \therefore c_1 = 5$</p> <p>$\frac{dy}{dx} = -2e^{-2x} + 5$</p> <p>$y = e^{-2x} + 5x(+c_2)$</p> <p>When $x = 2, y = e^{-4} \therefore c_2 = -10$</p> <p>$y = e^{-2x} + 5x - 10$</p>	<p>B1</p> <p>M1 A1</p> <p>B1 M1 $\sqrt{A1}$ [6]</p>	<p>B1 for $-2e^{-2x}$</p> <p>M1 for attempt to find c_1</p> <p>B1 for $-2e^{-2x}$ M1 for attempt to find c_2 $\sqrt{-2}$ times their c_1</p>
<p>9 (i) $2^5 + {}^5C_1 2^4(-3x) + {}^5C_2 2^3(-3x)^2$ $32 - 240x + 720x^2$</p> <p>(ii) $32a = 64, \quad a = 2$ $32b - 240a = -192,$ $b = 9$ $-240b + 720a = c$ $c = -720$</p>	<p>B1 B1 B1 [3]</p> <p>B1 M1 A1 M1 A1 [5]</p>	<p>B1 for 32 or 2^5 B1 for -240 B1 for 720.</p> <p>B1 for $a = 2$ M1 for equation in a and b equated to ± 192 A1 for $b = 9$ M1 for equation in a and b equated to c A1 for $c = -720$</p>
<p>10 (a) (i) $fg(x) = f\left(\frac{x}{x+2}\right)$ $= 3 - \frac{x}{x+2}$</p> <p>(ii) $3 - \frac{x}{x+2} = 10$ leading to $x = -1.75$</p> <p>(b) (i) $h(x) > 4$</p> <p>(ii) $h^{-1}(x) = e^{x-4}$ $h^{-1}(9) = e^5 \quad (\approx 148)$ or $4 + \ln x = 9,$ leading to $x = e^5$</p> <p>(iii) correct graphs</p>	<p>M1</p> <p>A1 [2]</p> <p>DM1 A1 [2]</p> <p>B1 [1]</p> <p>M1 A1 [2]</p> <p>B1 B1 B1 [3]</p>	<p>M1 for order</p> <p>DM1 for dealing with fractions sensibly</p> <p>M1 for attempting to obtain inverse function</p> <p>B1 for each curve B1 for idea of symmetry</p>

<p>11 (i) $\tan^2 2x = 3$ $\tan 2x = (\pm)\sqrt{3}$ $2x = 60^\circ, 120^\circ, 240^\circ, 300^\circ$ $x = 30^\circ, 60^\circ, 120^\circ, 150^\circ$</p> <p>(ii) $2\operatorname{cosec}^2 y + \operatorname{cosec} y - 3 = 0$ $(2\operatorname{cosec} y + 3)(\operatorname{cosec} y - 1) = 0$ $\operatorname{cosec} y = -\frac{3}{2}, 1$ $\sin y = -\frac{2}{3}, 1$ $y = 221.8^\circ, 318.2^\circ, y = 90^\circ$</p> <p>(iii) $\cos\left(z + \frac{\pi}{2}\right) = -\frac{1}{2}$ $z + \frac{\pi}{2} = \frac{2\pi}{3}, \frac{4\pi}{3}$ $z = \frac{\pi}{6}, \frac{5\pi}{6}$, allow 0.52, 2.62 rads</p>	<p>M1 DM1 A1, A1 [4]</p> <p>M1, A1 M1 A1, A1 [5]</p> <p>M1 A1, A1 [3]</p>	<p>M1 for an equation in $\tan^2 2x$ M1 for attempt to solve using $2x$ correctly A1 for any pair</p> <p>M1 for correct use of identity or other valid method A1 for a correct quadratic M1 for solution of quadratic and attempt to solve correctly A1 for $221.8^\circ, 318.2^\circ$, A1 for 90°</p> <p>M1 for dealing with sec and order of operations A1 for each</p>
<p>12</p> <p>(i) $\frac{dy}{dx} = \frac{(x+1)2x - x^2}{(x+1)^2}$ $= \frac{x(x+2)}{(x+1)^2}$ $\frac{dy}{dx} = 0, x = 0, -2$ $y = 0, -4$</p> <p>(ii) gradient of normal = $-\frac{4}{3}$ normal $y = -\frac{4}{3}x + \frac{11}{6}$, leads to $M (1.375, 0)$ $N (0, -4)$ Area = 2.75</p>	<p>M1 A1 DM1 A1, A1 [5]</p> <p>M1 A1 $\sqrt{}$ B1 B1 M1 $\sqrt{}$ A1 [6]</p>	<p>M1 for attempt to differentiate a quotient A1 correct allow unsimplified</p> <p>DM1 for equating to zero and an attempt to solve A1 for each pair (could be $x = 0$ and $x = -2$)</p> <p>M1 for attempt to obtain gradient of the normal A1 for a correct (unsimplified) normal equation Follow through on their normal B1 for N M1 for attempt to get area of triangle Ft on their M and N (must be on axes)</p>