UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS **4037/23**

Paper 2 **October/November 2010**

2 hours

Additional Materials: Answer Booklet/Paper
Graph paper (1 sheet)

3996780884

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Booklet/Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **7** printed pages and **1** blank page.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1} b + \binom{n}{2}a^{n-2} b^2 + \ldots + \binom{n}{r}a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

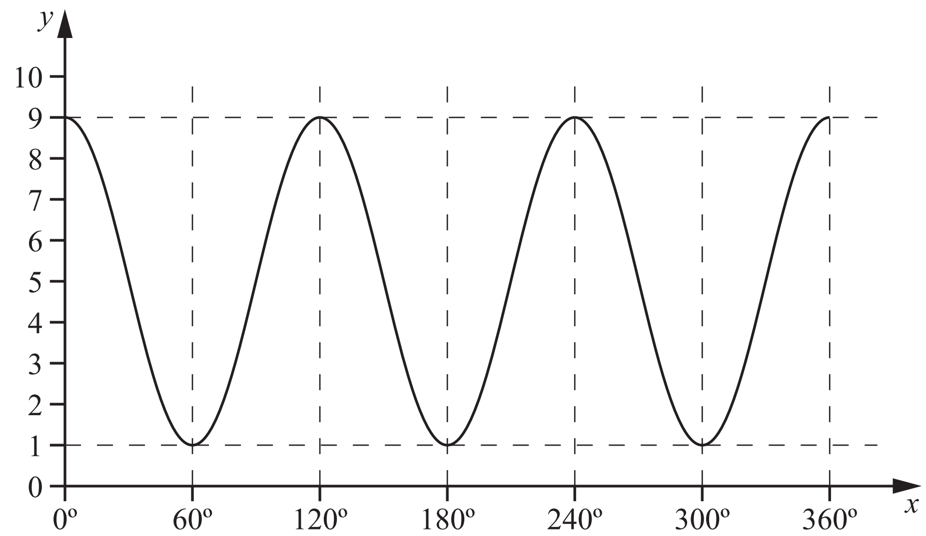
1 The two variables x and y are such that $y = \frac{10}{(x+4)^3}$.

(i) Find an expression for $\frac{dy}{dx}$. [2]

(ii) Hence find the approximate change in y as x increases from 6 to $6+p$, where p is small. [2]

2 Find the equation of the curve which passes through the point (4, 22) and for which $\frac{dy}{dx} = 3x(x-2)$. [4]

3 **(a)**



The diagram shows the curve $y = A\cos Bx + C$ for $0° \leqslant x \leqslant 360°$. Find the value of

(i) A, **(ii)** B, **(iii)** C. [3]

(b) Given that $f(x) = 6\sin 2x + 7$, state

(i) the period of f, [1]

(ii) the amplitude of f. [1]

4 **(i)** Find, in ascending powers of x, the first 4 terms of the expansion of $(1 + x)^6$. [2]

(ii) Hence find the coefficient of p^3 in the expansion of $(1 + p - p^2)^6$. [3]

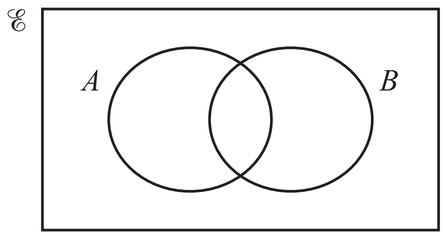
5 **(a)** Given that $\mathbf{A} = (2 \quad -4 \quad 1)$ and $\mathbf{B} = \begin{pmatrix} 3 & -1 \\ 0 & 5 \\ -2 & 7 \end{pmatrix}$, find the matrix product $\mathbf{AB}$. [2]

(b) Given that $\mathbf{C} = \begin{pmatrix} 3 & 5 \\ -2 & -4 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} 6 & -4 \\ 2 & 8 \end{pmatrix}$, find

(i) the inverse matrix $\mathbf{C}^{-1}$, [2]

(ii) the matrix $\mathbf{X}$ such that $\mathbf{CX} = \mathbf{D}$. [2]

6 **(a)**



Copy the diagram above and shade the region which represents the set $A' \cup B$. [1]

(b) The sets P, Q and R are such that

$$P \cap Q = \varnothing \text{ and } P \cup Q \subset R.$$

Draw a Venn diagram showing the sets P, Q and R. [2]

(c) In a group of 50 students F denotes the set of students who speak French and S denotes the set of students who speak Spanish. It is given that $n(F) = 24$, $n(S) = 18$, $n(F \cap S) = x$ and $n(F' \cap S') = 3x$. Write down an equation in x and hence find the number of students in the group who speak neither French nor Spanish. [3]

7 The line $y = 2x - 6$ meets the curve $4x^2 + 2xy - y^2 = 124$ at the points A and B. Find the length of the line AB. [7]

8 **(i)** Show that $(5 + 3\sqrt{2})^2 = 43 + 30\sqrt{2}$. [1]

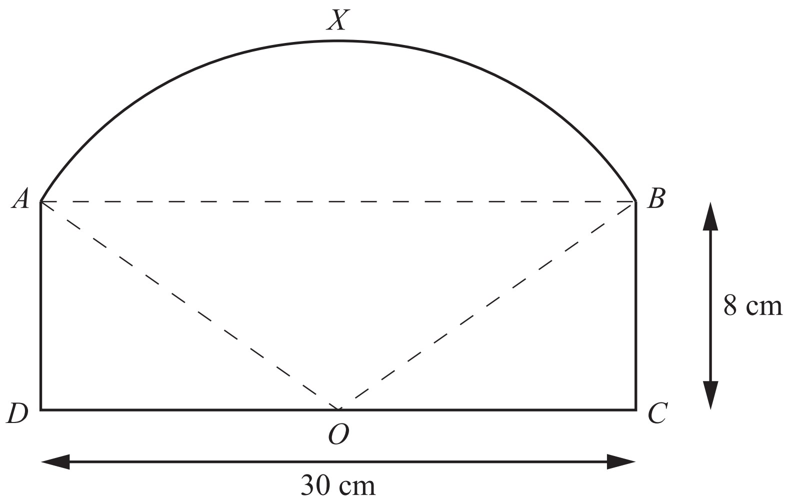
Hence find, **without using a calculator**, the positive square root of

(ii) $86 + 60\sqrt{2}$, giving your answer in the form $a + b\sqrt{2}$, where a and b are integers, [2]

(iii) $43 - 30\sqrt{2}$, giving your answer in the form $c + d\sqrt{2}$, where c and d are integers, [1]

(iv) $\dfrac{1}{43 + 30\sqrt{2}}$, giving your answer in the form $\dfrac{f + g\sqrt{2}}{h}$, where f, g and h are integers. [3]

9



The diagram shows a rectangle $ABCD$ and an arc AXB of a circle with centre at O, the mid-point of DC. The lengths of DC and BC are 30 cm and 8 cm respectively. Find

(i) the length of OA, [2]

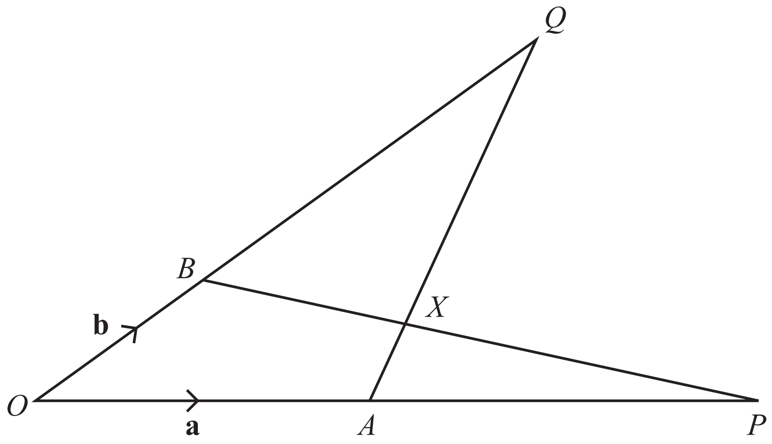
(ii) the angle AOB, in radians, [2]

(iii) the perimeter of figure $ADOCBXA$, [2]

(iv) the area of figure $ADOCBXA$. [2]

10 The equation of a curve is $y = x^2e^x$. The tangent to the curve at the point $P(1, e)$ meets the y-axis at the point A. The normal to the curve at P meets the x-axis at the point B. Find the area of the triangle OAB, where O is the origin. [9]

11



In the diagram $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{OP} = 2\mathbf{a}$ and $\overrightarrow{OQ} = 3\mathbf{b}$.

(i) Given that $\overrightarrow{AX} = \mu\overrightarrow{AQ}$, express $\overrightarrow{OX}$ in terms of μ, **a** and **b**. [3]

(ii) Given that $\overrightarrow{BX} = \lambda\overrightarrow{BP}$, express $\overrightarrow{OX}$ in terms of λ, **a** and **b**. [3]

(iii) Hence find the value of μ and of λ. [3]

12 Answer only **one** of the following two alternatives.

EITHER

The table shows values of the variables v and p which are related by the equation $p = \frac{a}{v^2} + \frac{b}{v}$, where a and b are constants.

v	2	4	6	8
p	6.22	2.84	1.83	1.35

(i) Using graph paper, plot $v^2 p$ on the y-axis against v on the x-axis and draw a straight line graph. [2]

(ii) Use your graph to estimate the value of a and of b. [4]

In another method of finding a and b from a straight line graph, $\frac{1}{v}$ is plotted along the x-axis. In this case, and without drawing a second graph,

(iii) state the variable that should be plotted on the y-axis, [2]

(iv) explain how the values of a and b could be obtained. [2]

OR

The table shows experimental values of two variables r and t.

t	2	8	24	54
r	22	134	560	1608

(i) Using the y-axis for $\ln r$ and the x-axis for $\ln t$, plot $\ln r$ against $\ln t$ to obtain a straight line graph. [2]

(ii) Find the gradient and the intercept on the y-axis of this graph and express r in terms of t. [6]

Another method of finding the relationship between r and t from a straight line graph is to plot $\lg r$ on the y-axis and $\lg t$ on the x-axis. Without drawing this second graph, find the value of the gradient and of the intercept on the y-axis for this graph. [2]

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