Cambridge O Level

**Cambridge International Examinations**
Cambridge Ordinary Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 2 6 4 4 7 1 5 5 2 7 *

**ADDITIONAL MATHEMATICS** **4037/22**

Paper 2 **May/June 2015**

**2 hours**

Candidates answer on the Question Paper.

No Additional Materials are required.

**READ THESE INSTRUCTIONS FIRST**

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **16** printed pages.

DC (KN/SW) 107048/1


CAMBRIDGE International Examinations

***Mathematical Formulae***

## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial Theorem*

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

*Formulae for $\Delta ABC$*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

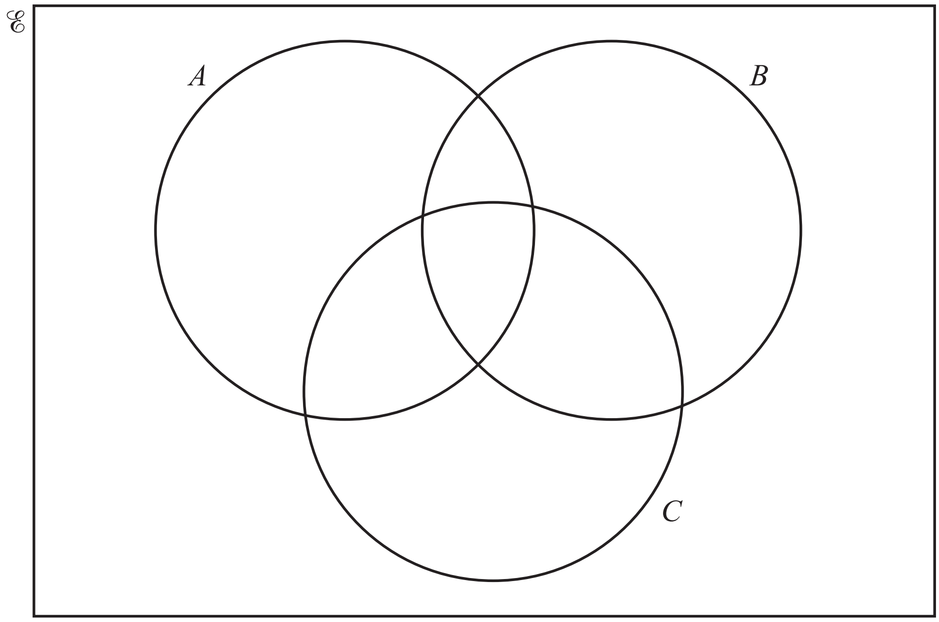
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

**1** The universal set contains all the integers from 0 to 12 inclusive. Given that

$A = \{1, 2, 3, 8, 12\}$, $B = \{0, 2, 3, 4, 6\}$ and $C = \{1, 2, 4, 6, 7, 9, 10\}$,

**(i)** complete the Venn diagram, [3]




**(ii)** state the value of $n(A' \cap B' \cap C)$, [1]

**(iii)** write down the elements of the set $A' \cap B \cap C$. [1]

**2** The table shows the number of passengers in Economy class and in Business class on 3 flights from London to Paris. The table also shows the departure times for the 3 flights and the cost of a single ticket in each class.

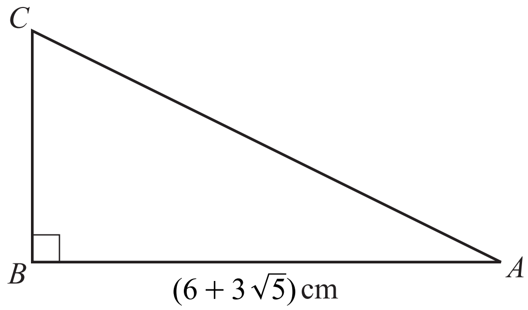
| Departure time | Number of passengers in Economy class | Number of passengers in Business class |
|---|---|---|
| 09 30 | 60 | 50 |
| 13 30 | 70 | 52 |
| 15 45 | 58 | 34 |
| Single ticket price (£) | 120 | 300 |

**(i)** Write down a matrix, **P**, for the numbers of passengers and a matrix, **Q**, of single ticket prices, such that the matrix product **QP** can be found. [2]

**(ii)** Find the matrix product **QP**. [2]

**(iii)** Given that $\mathbf{R} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, explain what information is found by evaluating the matrix product **QPR**. [1]

**3** **Do not use a calculator in this question.**




The diagram shows the right-angled triangle $ABC$, where $AB = (6+3\sqrt{5})$cm and angle $B = 90°$. The area of this triangle is $\left(\frac{36+15\sqrt{5}}{2}\right)$cm$^2$.

**(i)** Find the length of the side $BC$ in the form $(a+b\sqrt{5})$cm, where $a$ and $b$ are integers. [3]

**(ii)** Find $(AC)^2$ in the form $(c+d\sqrt{5})$cm$^2$, where $c$ and $d$ are integers. [2]

**4** A river, which is 80 m wide, flows at $2\,\text{ms}^{-1}$ between parallel, straight banks. A man wants to row his boat straight across the river and land on the other bank directly opposite his starting point. He is able to row his boat in still water at $3\,\text{ms}^{-1}$. Find

**(i)** the direction in which he must row his boat, [2]

**(ii)** the time it takes him to cross the river. [3]

**5** Solve the simultaneous equations

$$2x^2 + 3y^2 = 7xy,$$
$$x + y = 4.$$

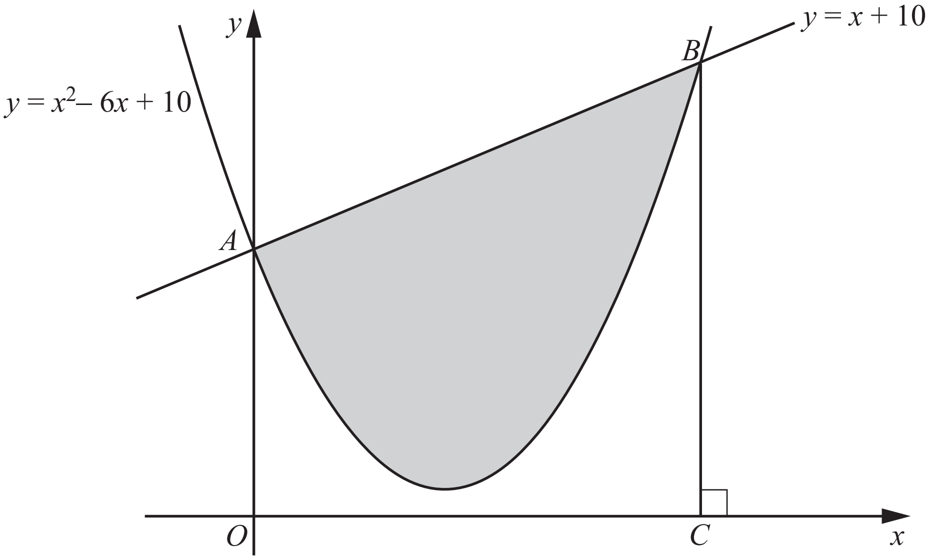
[5]

**6** **(a)** Solve $6^{x-2} = \frac{1}{4}$. [2]

**(b)** Solve $\log_a 2y^2 + \log_a 8 + \log_a 16y - \log_a 64y = 2\log_a 4$. [4]

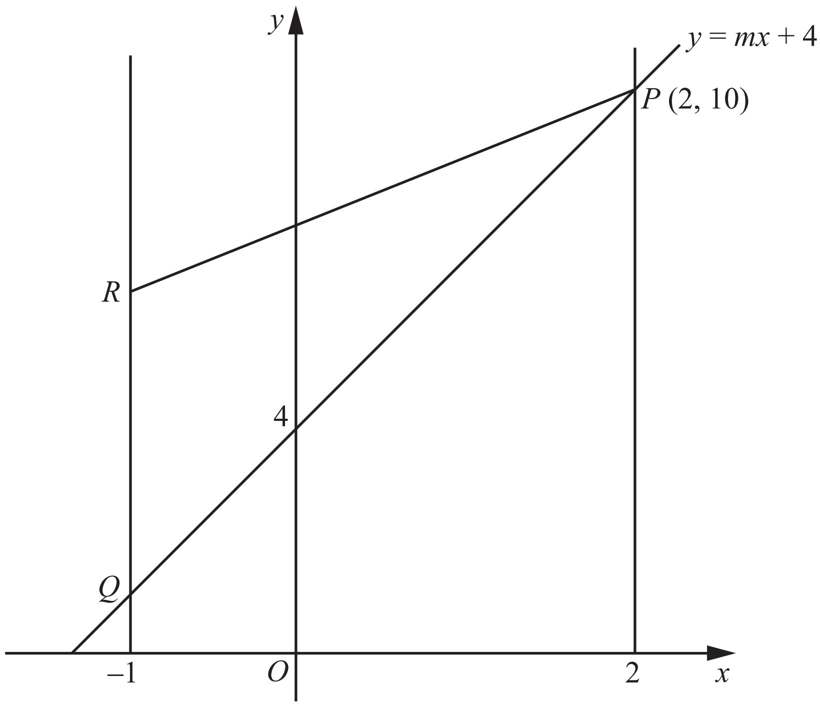
**7** In the expansion of $(1+2x)^n$, the coefficient of $x^4$ is ten times the coefficient of $x^2$. Find the value of the positive integer, $n$. [6]

**8**




The graph of $y = x^2 - 6x + 10$ cuts the $y$-axis at $A$. The graphs of $y = x^2 - 6x + 10$ and $y = x + 10$ cut one another at $A$ and $B$. The line $BC$ is perpendicular to the $x$-axis. Calculate the area of the shaded region enclosed by the curve and the line $AB$, showing all your working. [8]

**9 Solutions by accurate drawing will not be accepted.**




The line $y = mx + 4$ meets the lines $x = 2$ and $x = -1$ at the points $P$ and $Q$ respectively. The point $R$ is such that $QR$ is parallel to the $y$-axis and the gradient of $RP$ is 1. The point $P$ has coordinates (2, 10).

**(i)** Find the value of $m$. [2]

**(ii)** Find the $y$-coordinate of $Q$. [1]

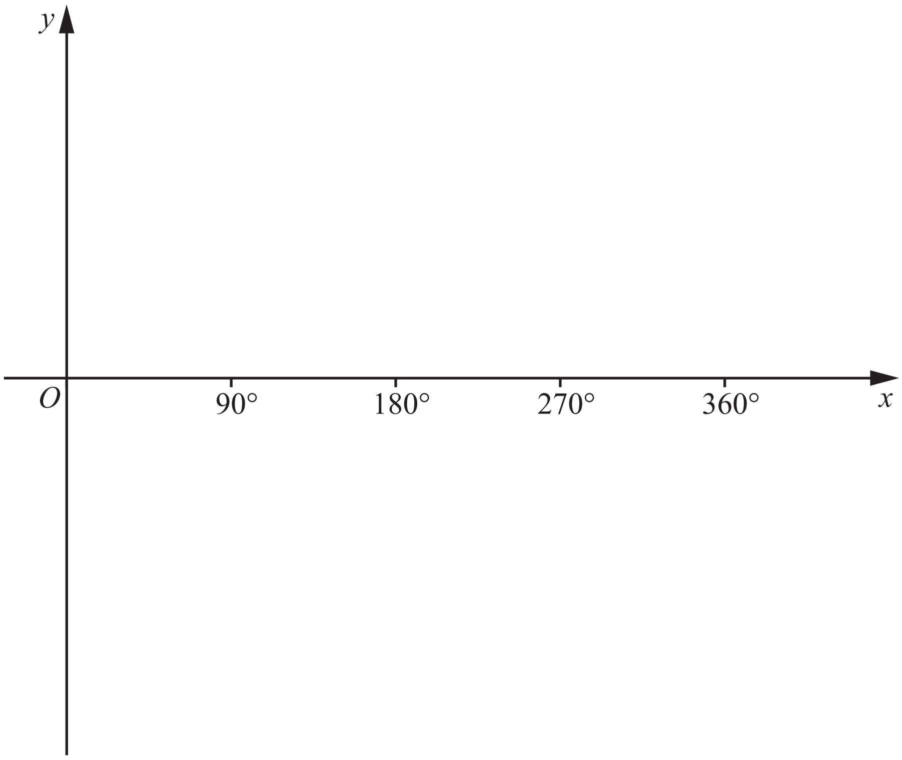
**(iii)** Find the coordinates of $R$. [2]

**(iv)** Find the equation of the line through $P$, perpendicular to $PQ$, giving your answer in the form $ax + by = c$, where $a$, $b$ and $c$ are integers. [3]

**(v)** Find the coordinates of the midpoint, $M$, of the line $PQ$. [2]

**(vi)** Find the area of triangle $QRM$. [2]

**10** **(a)** The function f is defined by $\mathrm{f}: x \mapsto |\sin x|$ for $0° \leqslant x \leqslant 360°$. On the axes below, sketch the graph of $y = \mathrm{f}(x)$. [2]

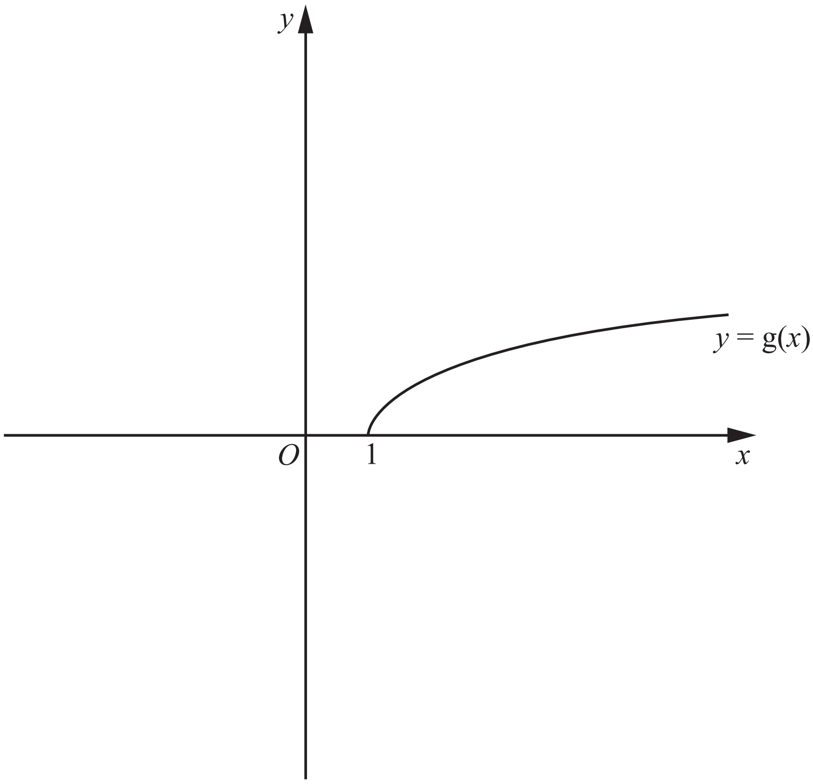



**(b)** The functions g and hg are defined, for $x \geqslant 1$, by

$$\mathrm{g}(x) = \ln(4x - 3),$$
$$\mathrm{hg}(x) = x.$$

**(i)** Show that $\mathrm{h}(x) = \dfrac{\mathrm{e}^x + 3}{4}$. [2]

**(ii)**

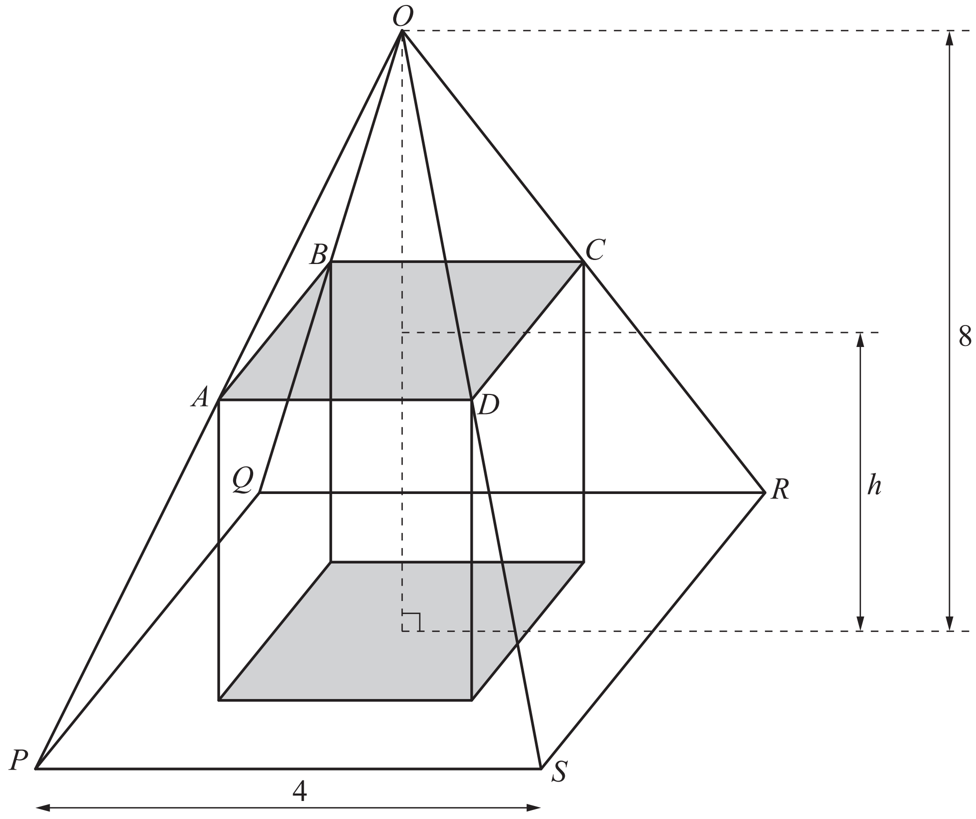



The diagram shows the graph of $y = \mathrm{g}(x)$. Given that g and h are inverse functions, sketch, on the same diagram, the graph of $y = \mathrm{h}(x)$. Give the coordinates of any point where your graph meets the coordinate axes. [2]

**(iii)** State the domain of h. [1]

**(iv)** State the range of h. [1]

**11**




The diagram shows a cuboid of height $h$ units inside a right pyramid $OPQRS$ of height 8 units and with square base of side 4 units. The base of the cuboid sits on the square base $PQRS$ of the pyramid. The points $A$, $B$, $C$ and $D$ are corners of the cuboid and lie on the edges $OP$, $OQ$, $OR$ and $OS$, respectively, of the pyramid $OPQRS$. The pyramids $OPQRS$ and $OABCD$ are similar.

**(i)** Find an expression for $AD$ in terms of $h$ and hence show that the volume $V$ of the cuboid is given by $\quad V = \dfrac{h^3}{4} - 4h^2 + 16h$ units$^3$. [4]

**(ii)** Given that $h$ can vary, find the value of $h$ for which $V$ is a maximum. [4]

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**Question 12 is printed on the next page.**

**12** **(i)** Show that $x = -2$ is a root of the polynomial equation $15x^3 + 26x^2 - 11x - 6 = 0$. [1]

**(ii)** Find the remainder when $15x^3 + 26x^2 - 11x - 6$ is divided by $x - 3$. [2]

**(iii)** Find the value of $p$ and of $q$ such that $15x^3 + 26x^2 - 11x - 6$ is a factor of $15x^4 + px^3 - 37x^2 + qx + 6$. [4]