

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Ordinary Level

MARK SCHEME for the May/June 2015 series

4037 ADDITIONAL MATHEMATICS

4037/22

Paper 2, maximum raw mark 80

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Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

1	(i)		B3,2,1,0	2 correctly placed in Venn diagram; 1, 3, 4, 6 correctly placed; 12, 8, 0, 7, 9, 10 correctly placed; 11, 5 correctly placed
	(ii)	3	B1ft	correct or correct ft <i>their</i> (i), provided non-zero
	(iii)	{4, 6}	B1ft	correct or correct ft <i>their</i> (i), provided not the empty set
2	(i)	$[\mathbf{P} =] \begin{pmatrix} 60 & 70 & 58 \\ 50 & 52 & 34 \end{pmatrix} \text{ and } [\mathbf{Q} =] (120 \quad 300)$	B2	or $[\mathbf{P} =] \begin{pmatrix} 50 & 52 & 34 \\ 60 & 70 & 58 \end{pmatrix} \text{ and}$ $[\mathbf{Q} =] (300 \quad 120)$ or B1 if one error may be written as an unevaluated product; B0 if choice of \mathbf{P} and \mathbf{Q} offered
	(ii)	(22200 24000 17160)	B2	must have brackets and must not have commas; must be a 1 by 3 matrix; must be from correct product; working may be seen in (i) or B1 for any two elements correct
	(iii)	The total (amount of revenue) from all (three) flights. oe	B1	do not accept, e.g. The total amount from each flight; must be a comment not just a figure; must not contain a contradiction

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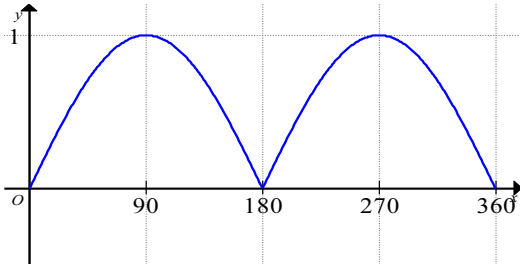
<p>3 (i)</p> $\frac{(36 + 15\sqrt{5})}{(6 + 3\sqrt{5})} \times \frac{(6 - 3\sqrt{5})}{(6 - 3\sqrt{5})} \text{ oe}$ $\frac{216 + 90\sqrt{5} - 108\sqrt{5} - 225}{-9}$ <p>$1 + 2\sqrt{5}$ cao</p> <p>Alternative method:</p> $36 + 15\sqrt{5} = (6a + 15b) + (3a + 6b)\sqrt{5}$ $6a + 15b = 36$ $3a + 6b = 15$ <p>$a = 1$ and $b = 2$</p> <p>(ii)</p> $\left[AC^2 = (6 + 3\sqrt{5})^2 + \text{their } (1 + 2\sqrt{5})^2 \right]$ $= 36 + 36\sqrt{5} + 45 + \text{their } (1 + 4\sqrt{5} + 20)$ <p>$102 + 40\sqrt{5}$ cao</p>		<p>M1</p> <p>DM1</p> <p>A1</p> <p>M1</p> <p>DM1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>or $\frac{(12 + 5\sqrt{5})}{(2 + \sqrt{5})} \times \frac{(2 - \sqrt{5})}{2 - \sqrt{5}}$ oe</p> <p>or $\frac{24 + 10\sqrt{5} - 12\sqrt{5} - 25}{-1}$</p> <p>or $-(24 + 10\sqrt{5}) - 12\sqrt{5} - 25$</p> <p>allow $a = 1$ and $b = 2$</p> <p>or $1 + 2\sqrt{5}$</p> <p>correct or correct ft expansions, using Pythagoras with $(6 + 3\sqrt{5})$ and <i>their</i> BC</p> <p>ignore attempts to square root after correct answer seen</p>
<p>4 (i)</p> $\cos(x) = \frac{2}{3} \text{ oe soi}$ <p>$48.189\dots^\circ$ or $131.810\dots^\circ$ or $0.8410\dots$ rad or $2.3(00\dots)$ rad oe isw</p> <p>with reference axis indicated by comment, e.g. “to the bank” or “upstream”, etc. or clearly marked on a diagram</p>		<p>M1</p> <p>A1</p>	<p>Alternatively</p> $\sin(y) = \frac{2}{3} \text{ oe soi}$ <p>$41.810\dots^\circ$ or $0.7297\dots$ or $0.73(0)$ rad oe isw</p> <p>with reference axis indicated by comment, e.g. “to the perpendicular with the bank”, etc. or clearly marked on a diagram</p> <p>If M0 then SC1 for an unsupported answer of $138.189\dots^\circ$ or $2.4118\dots$ rad or $318.189\dots^\circ$ or $5.5534\dots$ rad with reference axis indicated by comment, e.g. “on a bearing of” or “from North” or clearly marked on a diagram</p>

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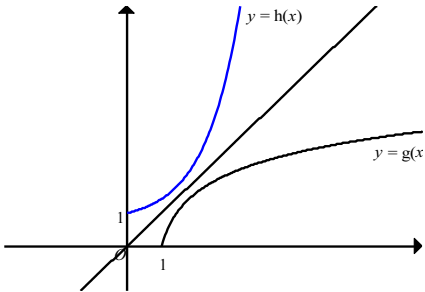
(ii)	<p>Speed = $\sqrt{9-4}$ ($=\sqrt{5}$) or $3 \sin 48.2$ or $2 \tan 48.2$ or $3 \cos 41.8$ or $\frac{2}{\tan 41.8}$ or $\sqrt{2^2 + 3^2 - 2 \times 2 \times 3 \cos 48.2}$ oe or 2.236(0...) rot to 4 or more figs or 2.24 [m/s] soi</p> <p>time = $\frac{80}{\text{their } \sqrt{5}}$ oe 35.66 to 35.8 (seconds) oe</p>	<p>B1</p> <p>M1</p> <p>A1</p>	<p>Or Distance = $\frac{80}{\sin 48.2} = 107.(33\dots)$ oe soi</p> <p>time = $\frac{\text{their } 107.33\dots}{3}$</p> <p>ignore subsequent rounding or attempted conversion to, e.g. minutes but A0 if answer spoiled by continuation of method</p> <p>if no working, so B0 M0, then allow B3 for an answer 35.66 to 35.8 oe</p>
5	<p>Substitution of either $4 - x$ or $4 - y$ into equation of curve and brackets expanded</p> <p>$12x^2 - 52x + 48 [= 0]$ or $12y^2 - 44y + 32 [= 0]$ oe</p> <p>Solve their 3-term quadratic $x = \frac{4}{3}$ and 3 isw $y = \frac{8}{3}$ and 1 isw</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>condone one sign error or slip in either equation of curve or expansion of brackets; condone omission of $= 0$, BUT $4 - x$ or $4 - y$ must be correct</p> <p>dep on a valid substitution attempt or $x = \frac{4}{3}$ $y = \frac{8}{3}$ not from wrong working</p> <p>or $x = 3$ $y = 1$ not from wrong working</p> <p>if no working, allow full marks for fully correct answer only.</p>
6 (a)	<p>$(x - 2) \log 6 = \log \left(\frac{1}{4} \right)$ oe or $\log_6 \left(\frac{1}{4} \right) = x - 2$ oe</p> <p>1.23 or 1.226(29...) rot to 4 or more figures isw</p>	<p>M1</p> <p>A1</p>	<p>or $x \log 6 = \log \left(\frac{36}{4} \right)$ oe</p> <p>or $x \log 6 - \log 36 = \log 1 - \log 4$ oe</p> <p>correct answer or 1.22 implies M1</p>

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<p>8</p>	<p>Method 1 (Separate areas subtracted)</p> <p>$[x_B = x_C =] 7$ soi</p> $\left[\int (x^2 - 6x + 10) dx = \right] \frac{x^3}{3} - \frac{6x^2}{2} + 10x$ <p>Correct or correct ft substitution of limits 0 and <i>their</i> 7 into <i>their</i> $\left[\frac{x^3}{3} - \frac{6x^2}{2} + 10x \right]$</p> $\frac{1}{2}(10+17) \times 7 \text{ oe or}$ $\int_0^7 (x+10) dx = \left[\frac{x^2}{2} + 10x \right]_0^7 = \frac{(7)^2}{2} + 10(7) \text{ oe}$ <p><i>their</i> $\left(\frac{189}{2} - \frac{112}{3} \right)$</p> $\frac{343}{6} \text{ or } 57\frac{1}{6} \text{ or } 57.2 \text{ to 3 sf or } 57.16(6\dots)$ <p>rot to 4 figs isw</p> <p>Method 2 (Subtracting and using integration once)</p> <p>$[x_B = x_C =] 7$ soi</p> $\int (-x^2 + 7x) dx$ $\left[-\frac{x^3}{3} + \frac{7x^2}{2} \right] \text{ oe or } \left[\frac{x^3}{3} - \frac{7x^2}{2} \right] \text{ oe}$ <p>Correct or correct ft substitution of limits 0 and <i>their</i> 7</p> <p>into <i>their</i> $\left[-\frac{x^3}{3} + \frac{7x^2}{2} \right]$</p> $\frac{343}{6} \text{ or } 57\frac{1}{6} \text{ or } 57.2 \text{ to 3 sf or } 57.16(6\dots)$ <p>rot to 4 figs isw</p>	<p>B1</p> <p>M2</p> <p>DM1</p> <p>B2</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>B1</p> <p>M3</p> <p>M2</p> <p>A1</p>	<p>or M1 for at least one term correct</p> <p>dep on at least M1 being earned; evidence of substitution must be seen in <i>their</i> integral which must be at least two terms; condone omission of lower limit;</p> <p>or M1 for $\frac{1}{2}(\text{their } 10 + \text{their } 17) \times \text{their } 7$ oe</p> <p>or B1 for $\int (x+10) dx = \frac{x^2}{2} + 10x$</p> <p>dep on a genuine attempt to integrate the equation of the curve; must be <i>their</i> area trapezium/under the line – <i>their</i> attempt at area under curve</p> <p>from full and correct working with no omitted steps</p> <p>condone omission of dx</p> <p>or M2 for $\int (px^2 + qx) dx = \frac{px^3}{3} + \frac{qx^2}{2}$ oe either with $p = \pm 1$ or $q = \pm 7$</p> <p>or M1 for $\int (px^2 + qx) dx = \frac{px^3}{3} + \frac{qx^2}{2}$ with non-zero constants p and q, with $p \neq \pm 1$ and $q \neq \pm 7$</p> <p>dep on a valid integration attempt; evidence of substitution must be seen; condone omission of lower limit;</p> <p>from full and correct working with no omitted steps</p>
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<p>9 (i)</p> <p>$10 = 2m + 4$ soi</p> <p>$m = 3$</p> <p>(ii) 1</p> <p>(iii) $\frac{10 - y_R}{2 - -1} = 1$ oe soi</p> <p>$(-1, 7)$ or $x = -1$ and $y = 7$</p> <p>(iv) Use of $m_1 m_2 = -1$ with <i>their</i> m from (i)</p> <p>$y - 10 = \left(\text{their} - \frac{1}{3} \right) (x - 2)$</p> <p>$3y + x = 32$ isw</p> <p>(v) $\left(\frac{1}{2}, \text{their} \frac{11}{2} \right)$ oe isw</p> <p>(vi) 4.5 oe cao</p>		<p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>B1, B1ft</p> <p>B2</p>	<p>or $[m =] \frac{10 - 4}{2 - 0}$ oe soi</p> <p>or $y = x + 8$ oe</p> <p>if $y = 7$ only stated, provided that $x = -1$ is soi in working allow both marks</p> <p>if M0 then B1 for $y = 7$ only with no working</p> <p>may be implied by perpendicular gradient seen in equation</p> <p>or $\left(\text{their} - \frac{1}{3} \right) x + c$ and</p> <p>$10 = \left(\text{their} - \frac{1}{3} \right) 2 + c$</p> <p>allow for correct equation with integer coefficients in any simplified form</p> <p>ft <i>their</i> y_Q</p> <p>or M1 for $\left(\frac{2-1}{2}, \frac{10+1}{2} \right)$ seen</p> <p>not from wrong working</p> <p>or M1 for any correct method with correct coordinates</p>
<p>10 (a)</p>		<p>B2, 1, 0</p> <p>correct sinusoidal/reflected sinusoidal shape, all above x-axis with intent to have all maximum points of equal height;</p> <p>2 maximum points of intended equal height only over 0 to 360;</p> <p>all max points clearly at $y = 1$;</p> <p>cusp at 180</p>	

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<p>(b)(i)</p> <p>$[hg(x) =] \frac{e^{\ln(4x-3)} + 3}{4}$</p> <p>fully correct and completion to $[hg(x) =] x$</p> <p>(ii)</p>  <p>(iii)</p> <p>$x \geq 0$ or $[0, \infty)$</p> <p>(iv)</p> <p>$y \geq 1$ or $[1, \infty)$</p>		<p>M1</p> <p>A1</p> <p>B2,1,0</p> <p>B1</p> <p>B1</p>	<p>Alternative method $y = \ln(4x - 3)$ and change of subject to x oe,</p> <p>fully correct and comment that $h(x) = g^{-1}(x)$ oe</p> <p>correct shape; 1 marked on the y-axis or $(0, 1)$ stated close by; curve with positive gradient in first quadrant only</p> <p>not domain ≥ 0</p> <p>or $h(x) \geq 1$, $h \geq 1$ etc.</p>
<p>11 (i)</p> <p>$\frac{8-h}{8}$ or $8 : 8 - h$ soi</p> <p>$\frac{8-h}{8} \times 4$ oe</p> <p>$h\left(\frac{8-h}{8} \times 4\right)^2$ oe</p> <p>expand and simplify to $\frac{h^3}{4} - 4h^2 + 16h$ AG</p> <p>(ii)</p> <p>$\frac{3}{4}h^2 - 8h + 16$ oe</p> <p>their $\left(\frac{3}{4}h^2 - 8h + 16\right) = 0$ and attempt to solve</p> <p>$\frac{8}{3}$ oe only</p>		<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A2</p>	<p>or $\frac{8}{8-h}$ or $8 - h : 8$ soi</p> <p>or $4 \div \frac{8}{8-h}$ oe</p> <p>h must be in the numerator of the expression for this mark;</p> <p>must be a 3-term quadratic; must be an attempt at a derivative</p> <p>or A1 for $h = \frac{8}{3}$ and 8</p> <p>allow 2.67 or 2.66(6...) rot to 4 or more figs for $\frac{8}{3}$</p>

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12	(i)	$-120 + 104 + 22 - 6 = 0$ or correct unsimplified form, e.g. $15(-2)^3 + 26(-2)^2 - 11(-2) - 6 = 0$ or $15(-8) + 26(4) - 11(-2) - 6 = 0$	B1	or correct synthetic division $\begin{array}{r rrrr} -2 & 15 & 26 & -11 & -6 \\ & & -30 & 8 & 6 \\ \hline & 15 & -4 & -3 & 0 \end{array}$
	(ii)	Substituting $x = 3$ into $15x^3 + 26x^2 - 11x - 6$ 600	M1	or correct synthetic division $\begin{array}{r rrrr} 3 & 15 & 26 & -11 & -6 \\ & & 45 & 213 & 606 \\ \hline & 15 & 71 & 202 & 600 \end{array}$
	(iii)	$(x - 1)(15x^3 + 26x^2 - 11x - 6)$ soi Multiply out $(x \pm 1)(15x^3 + 26x^2 - 11x - 6)$ and compare coefficients of x^3 or x to quartic $p = 11$ $q = 5$	A1 B1 M1 A1 A1	correct answer implies M1; must be explicitly identified as answer if using synthetic/long division methods by e.g. circling by inspection or division; may be implied by e.g. $(ax + b)(15x^3 + 26x^2 - 11x - 6)$ and $a = 1, b = -1$ seen in later work comparing coefficients or multiply out, e.g. $(ax + b)(15x^3 + 26x^2 - 11x - 6)$ and compare coefficients of x^3 or x to quartic correct p or q implies M1; correct p and q implies B1 M1